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Full Length Research Paper

Jacobi doubly periodic wave solutions for three versions of Benjamin-Bona- Mahony equation

M. A. Abdelkawy[1]*, M. A. Alghamdi[2] and A. H. Bhrawy[1,2]

[1]Department of Mathematics, Faculty of Science, Beni-Suef University, Beni-Suef, Egypt.
[2]Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia.

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A variety of exact solutions for the Kadomtsov-Petviashvili- Benjamin-Bona-Mahony (KP-BBM) equation, nonlinear Zakharov-Kuznetsov- Benjamin-Bona-Mahony (ZK-BBM) equation and the generalized ZK-BBM equation are developed by means of the extended Jacobi elliptic function expansion method. Soliton and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions.

Key words: Jacobi elliptic function method, soliton and triangular periodic solutions, nonlinear dispersive Kadomtsov-Petviashvili- Benjamin-Bona-Mahony (KP-BBM) equation, nonlinear Zakharov-Kuznetsov-Benjamin-Bona-Mahony (ZK-BBM) equation, the generalized ZK-BBM equation.

INTRODUCTION

The Benjamin-Bona-Mahony equation (BBME) was first introduced by Benjamin et al. (1972) as an improvement of the Korteweg-de Vries equation (KdV) for modeling long waves of small amplitude in 1+1 dimensions. They show the stability and uniqueness of solutions to the BBME equation. We study in this paper three versions of the BBME, they are the nonlinear dispersive Kadomtsov-Petviashvili-Benjamin-Bona-Mahony (KP-BBM) equation (Song et al., 2010; Wazwaz, 2005a, 2008a; Abdou, 2008a), nonlinear Zakharov-Kuznetsov–Benjamin-Bona-Mahony (ZK-BBM) equation and the generalized ZK-BBM equation (Wazwaz, 2005b; Abdou, 2007; Mahmoudi et al., 2008; Song and Yang, 2010; Wazwaz and Helal, 2005).

$$(u_t + u_x - au(u^2)_x + u_{xxt})_x + ku_{yy} = 0, \qquad (1)$$

$$u_t + u_x + au(u^2)_x + b(u_{xt} + u_{yy})_x = 0, \qquad (2)$$

$$u_t + u_x + au(u^3)_x + b(u_{xt} + u_{yy})_x = 0. \qquad (3)$$

*Corresponding author. E-mail: melkawy@yahoo.com

Nonlinear problems are of interest to engineers, physicists and mathematicians because most physical systems are inherently nonlinear in nature. Nonlinear partial differential equations (NPDEs) are difficult to solve and give rise to interesting phenomena such as chaos. The exact solutions of these NPDEs plays an important role in the study of nonlinear phenomena. In the past decades, many methods were developed for finding exact solutions of NPDEs such as Hirota's bilinear method (Wazwaz, 2008b), new similarity transformation method (Beavers and Denman, 1974), homogeneous balance method (Wang et al., 1996; Zhang, 2003), sine-cosine method (Wazwaz, 2006a; Tang et al., 2009), tanh function methods (Khater et al. 2010 Malfliet and Hereman, 1996; Wazwaz, 2006b), Riccati equations expansion method (Gepreel and Shehata, 2012; Liu et al., 2001), Exp-Function Method (Bhrawy et al., 2012; Mohyud-Din et al., 2010), Jacobi and Weierstrass elliptic function method (Liu et al., 2001; Zhao et al., 2006a b; Wen and Lü, 2009; Zhang and Xia, 2011)...etc.

In this paper, we extend the extended JEF method with symbolic computation to such special equations for constructing their interesting Jacobi doubly periodic wave solutions. It is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. In addition the algorithm that is used here is also a computerized method, in which an algebraic system is generated. Two key procedures and laborious to do by hand, but they can be implemented on a computer with the help of mathematica. The outputs of solving the algebraic system from a computer comprise a list of constants. In general if any of the parameters is left unspecified. We only consider the expansion in terms of the Jacobi functions $sn\xi$ and $cn\xi$. Further studies show that different Jacobi function expansions may lead to new periodic wave solutions.

MATERIALS AND METHODS

Extended Jacobi elliptic function method

Here, we introduce a simple description of the extended JEF method, for a given partial differential equation

$$G(u,u_x,u_y,u_t,u_{xy},...)=0. \quad (4)$$

We like to know whether travelling waves (or stationary waves) are solutions of Equation (4). The first step is to unite the independent variables x, y and t into one particular variable through the new variable.

$$\zeta = x+y+\nu t, \quad u(x,y,t)=U(\zeta),$$

where ν is the wave speed, reduce Equation (4) to an ordinary differential equation (ODE)

$$G(U,U',U'',U''',...)=0. \quad (5)$$

Our main goal is to find exact or at least approximate solutions, if possible, for this ODE. For this purpose, using the extended Jacobi elliptic function expansion method, $U(\zeta)$ can be expressed as a finite series of JEF, sn ζ,

$$u(x,y)=U(\zeta)=\sum_{i=0}^{N} a_i sn(\zeta)^i + \sum_{i=1}^{N} a_{-i} sn(\zeta)^{-i}. \quad (6)$$

The parameter N is determined by balancing the linear term(s) of highest order with the nonlinear one(s). And

$$cn^2(\zeta)=1-sn^2(\zeta), \qquad dn^2(\zeta)=1-m^2 sn^2(\zeta), \quad (7)$$

$$\frac{d}{d\zeta}sn\zeta = cn\zeta dn\zeta, \quad \frac{d}{d\zeta}cn\zeta = -sn\zeta dn\zeta, \quad \frac{d}{d\zeta}dn\zeta = -m^2 sn\zeta cn\zeta, \quad (8)$$

where $cn\zeta$ and $dn\zeta$ are the Jacobi elliptic cosine function and the JEF of the third kind, respectively, with the modulus m $(0<m<1)$. Since the highest degree of $\frac{d^pU}{d\zeta^p}$ is taken as

$$O\left(\frac{d^pU}{d\zeta^p}\right)=N+p, \qquad p=1,2,3,\cdots, \quad (9)$$

$$O\left(U^q\frac{d^pU}{d\zeta^p}\right)=(q+1)N+p, \quad q=0,1,2,\cdots, p=1,2,3,\cdots. \quad (10)$$

Normally N is a positive integer, so that an analytic solution in closed form may be obtained. Substituting Equations (6) to (10) into Equation (5) and comparing the coefficients of each power of $sn\zeta$ in both sides, to get an over-determined system of nonlinear algebraic equations with respect to ν, a_i and a_{-i}, $i=1,\cdots,N$. Solving the over-determined system of nonlinear algebraic equations by use of Mathematica. We can get other kinds of Jacobi doubly periodic wave solutions.

When $m \to 1$, the Jacobi functions degenerate to the hyperbolic functions,

$$sn\zeta \to tanh\zeta, \quad cn\zeta \to sech\zeta \quad \text{and} \quad dn\zeta \to sech\zeta.$$

When $m \to 0$, the Jacobi functions degenerate to the triangular functions,

$$sn\zeta \to sin\zeta, \quad cn\zeta \to cos\zeta \quad \text{and} \quad dn \to 1.$$

RESULTS

Kadomtsov-Petviashvili-Benjamin-Bona-Mahony equation

We first consider the KP-BBME in the following form:

$$(u_t+u_x-au(u^2)_x+u_{xxt})_x+ku_{yy}=0. \quad (11)$$

If we use the transformations

$$u(x,t)=U(\zeta), \qquad \zeta = x+\nu t. \quad (12)$$

It carries Equation (11) to the ODE.

$$((1+\nu)U'-a(U^2)'-b\nu U''')'+kU''=0. \quad (13)$$

Where by integrating twice we obtain, upon setting the constant of integration to zero,

$$(k+\nu+1)U-b\nu U''-aU^2=0. \quad (14)$$

Balancing the term U'' with the term U^2 we obtain $N=2$ then

$$U(\zeta)=\sum_{i=0}^{2} a_i sn^i(\zeta)+\sum_{i=1}^{2} a_{-i}(sn(\zeta))^{-i}. \quad (15)$$

Substituting Equation (15) into (14) and comparing the coefficients of each power of $sn(\zeta)$ in both sides, getting an over-determined system of nonlinear algebraic equations with respect to ν, a_i; $i=0,\ 1,\ -1,\ 2,\ -2$. Solving the over-determined system of nonlinear algebraic equations using Mathematica, we obtain three groups of constants:

$$a_{-1}=a_2=a_1=0,\quad a_{-2}=\ \pm\frac{3(1+k+\nu)}{2a\sqrt{1-m^2+m^4}},\quad a_0=\frac{(1+k+\nu)(\sqrt{1-m^2+m^4}\pm(1+m^2))}{2a\sqrt{1-m^2+m^4}}$$
$$\text{and}\ \ b=\mp\frac{(1+k+\nu)}{4\nu\sqrt{1-m^2+m^4}} \tag{16}$$

$$a_{-1}=a_{-2}=a_1=0,\quad a_2=\ \pm\frac{3m^2(1+k+\nu)}{2a\sqrt{1-m^2+m^4}},\quad a_0=\frac{(1+k+\nu)(\sqrt{1-m^2+m^4}\pm(1+m^2))}{2a\sqrt{1-m^2+m^4}}$$
$$\text{and}\ \ b=\mp\frac{(1+k+\nu)}{4\nu\sqrt{1-m^2+m^4}} \tag{17}$$

$$a_{-1}=a_1=0,\quad a_2=\ \pm\frac{3m^2(1+k+\nu)}{2a\sqrt{1+14m^2+m^4}},\quad a_0=\frac{(1+k+\nu)(\sqrt{1+14m^2+m^4}\pm(1+m^2))}{2a\sqrt{1-m^2+m^4}}$$
$$a_{-2}=\ \pm\frac{3(1+k+\nu)}{2a\sqrt{1+14m^2+m^4}}\quad\text{and}\ \ b=\mp\frac{(1+k+\nu)}{4\nu\sqrt{1+14m^2+m^4}}. \tag{18}$$

We find the following solutions of the ordinary differential Equation (14)

$$U_1=\frac{(1+k+\nu)(\sqrt{1-m^2+m^4}\pm(1+m^2))}{2a\sqrt{1-m^2+m^4}}\pm\frac{3(1+k+\nu)}{2a\sqrt{1-m^2+m^4}}(sn\zeta)^{-2}, \tag{19}$$

$$U_2=\frac{(1+k+\nu)(\sqrt{1-m^2+m^4}\pm(1+m^2))}{2a\sqrt{1-m^2+m^4}}\pm\frac{3m^2(1+k+\nu)}{2a\sqrt{1-m^2+m^4}}(sn\zeta)^{2}, \tag{20}$$

$$U_3=\frac{(1+k+\nu)(\sqrt{1+14m^2+m^4}\pm(1+m^2))}{2a\sqrt{1-m^2+m^4}}\pm\frac{3(1+k+\nu)}{2a\sqrt{1+14m^2+m^4}}[m^2(sn\zeta)^2+(sn\zeta)^{-2}]. \tag{21}$$

Then the solutions of the KP-BBME (11) are:

$$u_1=\frac{(1+k+\nu)(1+m^2+\sqrt{1-m^2+m^4})}{2a\sqrt{1-m^2+m^4}}\pm\frac{3(1+k+\nu)}{2a\sqrt{1-m^2+m^4}}(sn(x+y+\nu t))^{-2}, \tag{22}$$

$$u_2=\frac{(1+k+\nu)(1+m^2+\sqrt{1-m^2+m^4})}{2a\sqrt{1-m^2+m^4}}\pm\frac{3m^2(1+k+\nu)}{2a\sqrt{1-m^2+m^4}}(sn(x+y+\nu t))^{2}, \tag{23}$$

$$u_3=\frac{(1+k+\nu)(\sqrt{1+14m^2+m^4}\pm(1+m^2))}{2a\sqrt{1-m^2+m^4}}$$
$$\pm\frac{3(1+k+\nu)}{2a\sqrt{1+14m^2+m^4}}[m^2(sn(x+y+\nu t))^2+(sn(x+y+\nu t))^{-2}]. \tag{24}$$

The modulus of solitary wave solution u_1 (Equation 22) and u_2 (Equation 23) are displayed in Figures 1 and 2 respectively, with values of parameters listed in their captions.

Zakharov-Kuznetsov-Benjamin-Bona-Mahony equation

We consider the ZK-BBME in the following form:

$$u_t + u_x + au(u^2)_x + b(u_{xt} + u_{yy})_x = 0. \quad (25)$$

If we use the transformations

$$u(x,t) = U(\zeta), \qquad \zeta = x + \nu t. \quad (26)$$

It carries Equation (25) to the ODE

$$(1+\nu)U' + a(U^2)' + b(1+\nu)U''' = 0. \quad (27)$$

Where by integrating once we obtain, upon setting the constant of integration to zero,

$$(1+\nu)U + aU^2 + b(1+\nu)U'' = 0. \quad (28)$$

Balancing the term U'' with the term U^2 we obtain $N = 2$ then

$$U(\zeta) = \sum_{i=0}^{2} a_i sn^i(\zeta) + \sum_{i=1}^{2} a_{-i}(sn(\zeta))^{-i}. \quad (29)$$

Proceeding as in the previous case we obtain

$$a_{-1} = a_2 = a_1 = 0, \quad a_{-2} = \pm\frac{3(1+\nu)}{2a\sqrt{1-m^2+m^4}}, \quad a_0 = -\frac{(1+\nu)(\sqrt{1-m^2+m^4} \pm (1+m^2))}{2a\sqrt{1-m^2+m^4}}$$
$$\text{and} \quad b = \mp\frac{1}{4\nu\sqrt{1-m^2+m^4}} \quad (30)$$

$$a_{-1} = a_{-2} = a_1 = 0, \quad a_2 = \pm\frac{3m^2(1+\nu)}{2a\sqrt{1-m^2+m^4}}, \quad a_0 = -\frac{(1+\nu)(\sqrt{1-m^2+m^4} \pm (1+m^2))}{2a\sqrt{1-m^2+m^4}}$$
$$\text{and} \quad b = \mp\frac{1}{4\nu\sqrt{1-m^2+m^4}} \quad (31)$$

$$a_{-1} = a_1 = 0, \quad a_2 = \pm\frac{3m^2(1+\nu)}{2a\sqrt{1+14m^2+m^4}}, \quad a_0 = -\frac{(1+\nu)(\sqrt{1+14m^2+m^4} \pm (1+m^2))}{2a\sqrt{1-m^2+m^4}}$$
$$a_{-2} = \pm\frac{3(1+\nu)}{2a\sqrt{1+14m^2+m^4}} \quad \text{and} \quad b = \mp\frac{1}{4\nu\sqrt{1+14m^2+m^4}}. \quad (32)$$

We find the following solutions of Equation (28)

$$U_1 = \frac{(1+\nu)(\sqrt{1-m^2+m^4} \pm (1+m^2))}{2a\sqrt{1-m^2+m^4}} \pm \frac{3(1+\nu)}{2a\sqrt{1-m^2+m^4}}(sn\zeta)^{-2}, \quad (33)$$

$$U_2 = \frac{(1+\nu)(\sqrt{1-m^2+m^4} \pm (1+m^2))}{2a\sqrt{1-m^2+m^4}} \pm \frac{3m^2(1+\nu)}{2a\sqrt{1-m^2+m^4}}(sn\zeta)^{2}, \quad (34)$$

$$U_3 = \frac{(1+\nu)(\sqrt{1+14m^2+m^4} \pm (1+m^2))}{2a\sqrt{1-m^2+m^4}} \pm \frac{3(1+\nu)}{2a\sqrt{1+14m^2+m^4}}[m^2(sn\zeta)^2 + (sn\zeta)^{-2}]. \quad (35)$$

Then the solutions of the ZK-BBME (25) are:

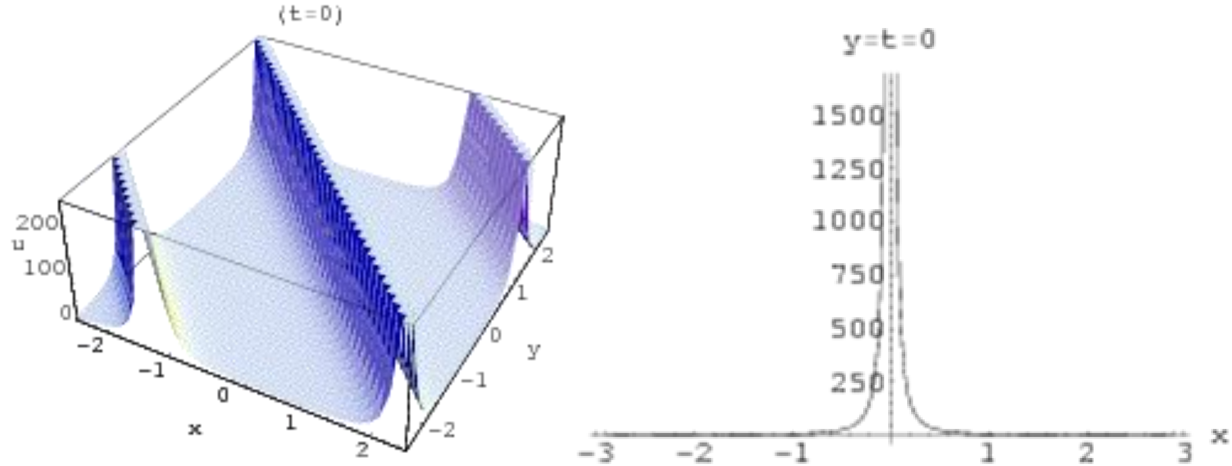

Figure 1. The modulus of solitary wave solution u_1 (Equation 22) where $m=\nu=k=a=0.5$.

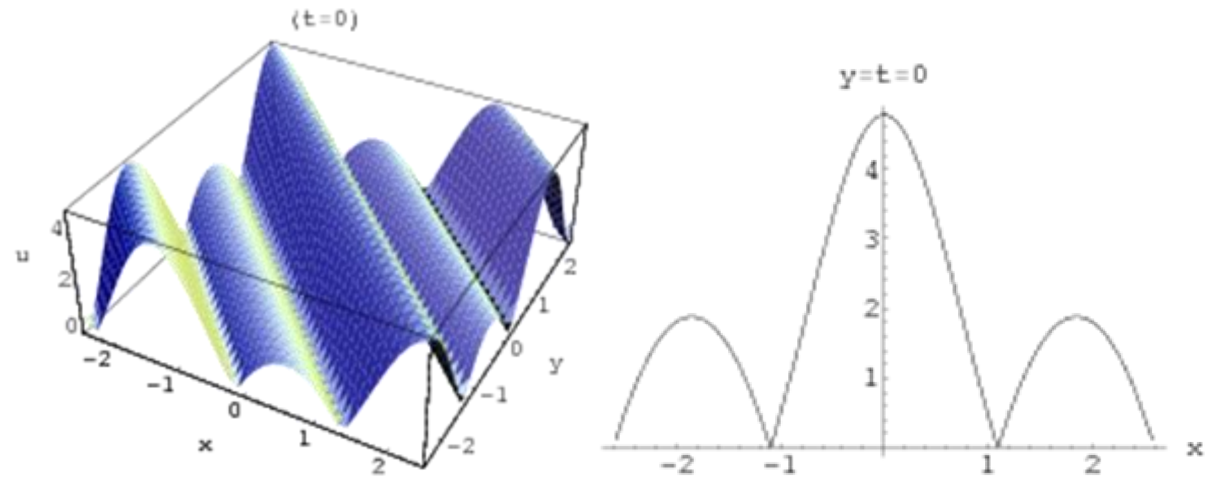

Figure 2. The modulus of solitary wave solution u_2 (Equation 23) where $m=\nu=k=a=0.5$.

$$u_1=\frac{(1+\nu)(1+m^2+\sqrt{1-m^2+m^4})}{2a\sqrt{1-m^2+m^4}}\pm\frac{3(1+\nu)}{2a\sqrt{1-m^2+m^4}}(sn(x+y+\nu t))^{-2}, \tag{36}$$

$$u_2=\frac{(1+\nu)(1+m^2+\sqrt{1-m^2+m^4})}{2a\sqrt{1-m^2+m^4}}\pm\frac{3m^2(1+\nu)}{2a\sqrt{1-m^2+m^4}}(sn(x+y+\nu t))^{2}, \tag{37}$$

$$u_3=\frac{(1+\nu)(\sqrt{1+14m^2+m^4}\pm(1+m^2))}{2a\sqrt{1-m^2+m^4}}\pm\frac{3(1+\nu)}{2a\sqrt{1+14m^2+m^4}}[m^2(sn(x+y+\nu t))^2+(sn(x+y+\nu t))^{-2}]. \tag{38}$$

Generalized Zakharov-Kuznetsov–Benjamin-Bona-Mahony equation

We consider the GZK-BBME in the following form:

$$u_t+u_x+au(u^3)_x+b(u_{xt}+u_{yy})_x=0. \tag{39}$$

If we use the transformations

$$u(x,t)=U(\zeta), \qquad \zeta = x+\nu t. \tag{40}$$

It carries Equation (39) to the ODE

$$(1+\nu)U' + a(U^3)' + b(1+\nu)U''' = 0. \tag{41}$$

Where by integrating once we obtain, upon setting the constant of integration to zero,

$$(1+\nu)U + aU^3 + b(1+\nu)U'' = 0. \tag{42}$$

Balancing the term U'' with the term U^3 we obtain $N=1$ then

$$U(\zeta)=\sum_{i=0}^{1} a_i sn^i(\zeta)+\sum_{i=1}^{1} a_{-i}(sn(\zeta))^{-i}. \tag{43}$$

Proceeding as in the previous case we obtain

$$a_{-1}=a_0=0, \quad a_1=\pm im\frac{2(1+\nu)}{\sqrt{a(1+m^2)}} \quad \text{and} \quad b=\frac{1}{1+m^2} \tag{44}$$

$$a_0=0, \quad a_1=\pm im\frac{2(1+\nu)}{\sqrt{a(1+(m-6)m)}}, \quad a_{-1}=\mp i\frac{2(1+\nu)}{\sqrt{a(1+(m-6)m)}} \quad \text{and} \quad b=\frac{1}{1+(m-6)m} \tag{45}$$

$$a_0=0, \quad a_1=\pm im\frac{2(1+\nu)}{\sqrt{a(1+(m+6)m)}}, \quad a_{-1}=\pm i\frac{2(1+\nu)}{\sqrt{a(1+(m+6)m)}} \quad \text{and} \quad b=\frac{1}{1+(m+6)m} \tag{46}$$

We find the following solutions of Equation (42)

$$U_1=\pm im\frac{2(1+\nu)}{\sqrt{a(1+m^2)}}sn\zeta, \tag{47}$$

$$U_2=\mp i\frac{2(1+\nu)}{\sqrt{a(1+(m-6)m)}}[msn\zeta-(sn\zeta)^{-1}]. \tag{48}$$

$$U_3=\mp i\frac{2(1+\nu)}{\sqrt{a(1+(m+6)m)}}[msn\zeta+(sn\zeta)^{-1}]. \tag{49}$$

Then the solutions of the GZK-BBME (39) are:

$$u_1=\pm im\frac{2(1+\nu)}{\sqrt{a(1+m^2)}}sn(x+y+\nu t), \tag{50}$$

$$u_2=\mp i\frac{2(1+\nu)}{\sqrt{a(1+(m-6)m)}}[msn(x+y+\nu t)-(sn(x+y+\nu t))^{-1}]. \tag{51}$$

$$u_3=\mp i\frac{2(1+\nu)}{\sqrt{a(1+(m+6)m)}}[msn(x+y+\nu t)+(sn(x+y+\nu t))^{-1}]. \tag{52}$$

DISCUSSION

The investigation of exact solutions is the key of understanding the nonlinear physical phenomena. It is known that many physical phenomena are often described by NLPDEs. Many methods for obtaining exact travelling solitary wave solutions to NLPDEs have been proposed. By introducing appropriate transformations and using extended Jacobi elliptic function expansion method, we have been able to obtain in a unified way with the aid of symbolic computation system-mathematica, a series of solutions including single and the combined Jacobi elliptic function. Moreover, it is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. For $m\to 1$, the above solutions are obtianed using the hyperbolic and extended hyperbolic functions method. Where $m\to 0$, these solutions are equivalent to these obtianed using the triangular and etended triangular functions method.

Conclusion

We extend the extended JEF method with symbolic computation to three equations for constructing their interesting Jacobi doubly periodic wave solutions. It is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. When $m\to 1$, the Jacobi functions degenerate to the hyperbolic functions and given the solutions by the extended hyperbolic functions methods. When $m\to 0$, the Jacobi functions degenerate to the triangular functions and the solutions given by extended triangular functions methods. Moreover, we can find a several solutions by replacing $sn\zeta$ in the expansion (6) with other kinds of Jacobi functions and repeating the same process as before.

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